Reg No.:_______________ Name:__________________________

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

Third Semester B.Tech Degree Examination December 2020 (2019 Scheme)

**Course Code: MAT201**

**Course Name: Partial Differential equations and Complex analysis**

Max. Marks: 100 Duration: 3 Hours

**PART A**

***Answer all questions. Each question carries 3 marks*** Marks

| | | |
|---|---|---|
| 1 | Derive a partial differential equation from the relation z = (x+ y) f ( $x^2$ - $y^2$ ) | (3) |
| 2 | Solve using direct integration $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} cosx$ | (3) |
| 3 | Solve 2z=xp +yq. | (3) |
| 4 | Write any three assumptions in deriving one dimensional heat equation. | (3) |
| 5 | Show that an analytic function f(z) = u+ iv is constant if its real part is constant. | (3) |
| 6 | Show that the function u = sin x cos hy is harmonic. | (3) |
| 7 | Find the Maclaurin series of f(z) = sinz | (3) |
| 8 | Evaluate $\oint_C \ln z \, dz$ , where C is the unit circle $\lvert z \rvert = 1$. | (3) |
| 9 | Find all singular points and residue of the function cosec z | (3) |
| 10 | Determine the location and order of zeros of the function $sin^4 (\frac{z}{2})$ | (3) |

**PART B**

***Answer any one full question from each module. Each question carries 14 marks***

**Module 1**

| | | |
|---|---|---|
| 11 (a) | Form the Partial differential equation by eliminating the arbitrary constants from $(x-a)^2 + (y-b)^2 = z^2 cot^2 \alpha$ | (5) |
| (b) | Solve 2xz –p $x^2$ - 2qxy + pq = 0 | (9) |
| 12 (a) | Solve $\frac{\partial^3 z}{\partial^2 x \, \partial y}$ = Cos ( 2x+ 3 y ) | (7) |
| (b) | Solve $x^2$ ( y – z )p + $y^2$ ( z- x ) q = $z^2$ ( x – y ) | (7) |

**Module 2**

| | | |
|---|---|---|
| 13 (a) | Derive the solution of the one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ using variable separable method. | (6) |
| (b) | An insulated rod of length l has its ends A and B maintained at $0^0$ C and | |

| | | | |
|---|---|---|---|
| | | $100^0$ C respectively until steady state conditions prevail. If B is suddenly reduced to $0^0$ C and maintained at $0^0$ C, find the temperature at a distance x from A at time t. | (8) |
| 14 | (a) | Derive the one dimensional heat flow equation. | (6) |
| | (b) | A tightly stretched string of length $l$ with fixed ends is initially in equilibrium position .If it is set vibrating by giving each points a velocity $v_0 sin^3(\frac{\pi x}{l})$. Find the displacement y(x,t). | (8) |

**Module 3**

| | | | |
|---|---|---|---|
| 15 | (a) | Find an analytic function whose real part is u = sinx coshy | (7) |
| | (b) | Find the image of the strip $\frac{1}{2} \leq x \leq 1$ under the transformation w = $z^2$ | (7) |
| 16 | (a) | Check whether w = logz is analytic. | (8) |
| | (b) | Show that under the transformation w = $\frac{1}{z}$, the circle $x^2 + y^2 - 6x = 0$ is transformed into a straight line in the W plane. | (6) |

**Module 4**

| | | | |
|---|---|---|---|
| 17 | (a) | Integrate counter clockwise around the unit circle $\oint_C \frac{\sin 2z}{z^4}$ dz | (7) |
| | (b) | Find the Taylor series of $\frac{1}{1+z}$ about the centre $z_0$ = i | (7) |
| 18 | (a) | Evaluate $\int_0^{1+i}(x - y + ix^2)$ dz along the parabola y= $x^2$. | (7) |
| | (b) | Evaluate $\oint_c \frac{\log z}{(z-4)^2}$ dz counter clockwise around the circle $\lvert z - 3\rvert$=2. | (7) |

**Module 5**

| | | | |
|---|---|---|---|
| 19 | (a) | Find the Laurent's series expansion of $\frac{z^2 - 1}{z^2 - 5z + 6}$ about z= 0 in the region 2< $\lvert z\rvert < 3$ | (5) |
| | (b) | Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - cos\theta}$. | (9) |
| 20 | (a) | Evaluate $\oint_C \frac{z-23}{z^2 - 4z - 5}$ dz where C : $\lvert z - 2 - i\rvert$ = 3.2 using Residue theorem. | (5) |
| | (b) | Evaluate $\int_0^{\infty} \frac{(x^2 + 2)dx}{(x^2 + 1)(x^2 + 4)}$. | (9) |

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