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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIFTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS (CS)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries3 marks. Marks

1 Consider a graph G with 4 vertices: v1, v2, v3 and v4 and the degrees of vertices are 3, 5, 2 and 1 respectively. Is it possible to construct such a graph G? If not, why? (3)

2 Draw a disconnected simple graph G1 with 10 vertices and 4 components and also calculate the maximum number of edges possible in G1. (3)

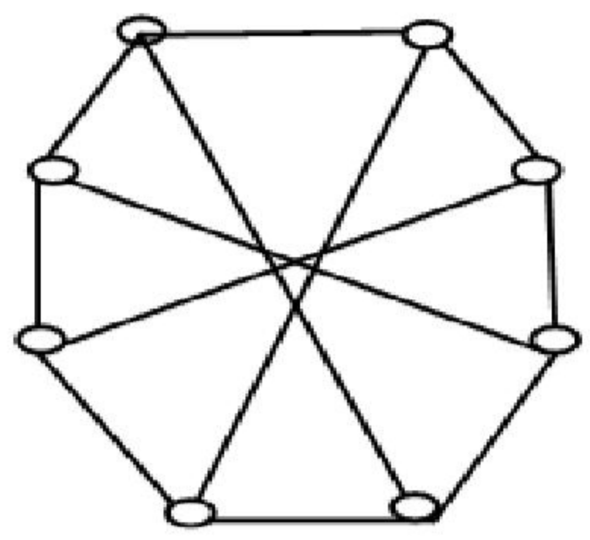
3 State Dirac's theorem for hamiltonicity and why it is not a necessary condition for a simple graph to have a Hamiltonian circuit. (3)

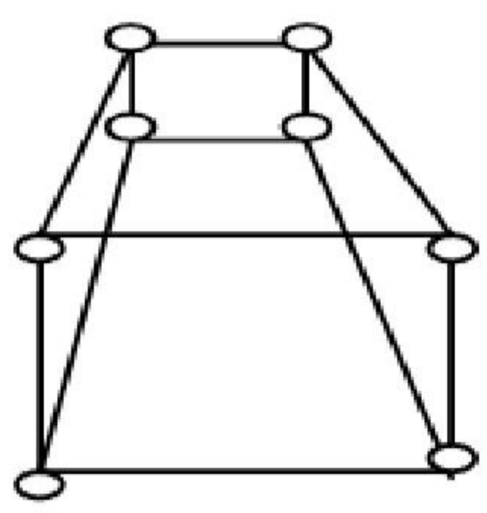
4 Differentiate between symmetric and asymmetric digraphs with examples and draw a complete symmetric digraph of four vertices. (3)

PART B

Answer any two full questions, each carries 9 marks.

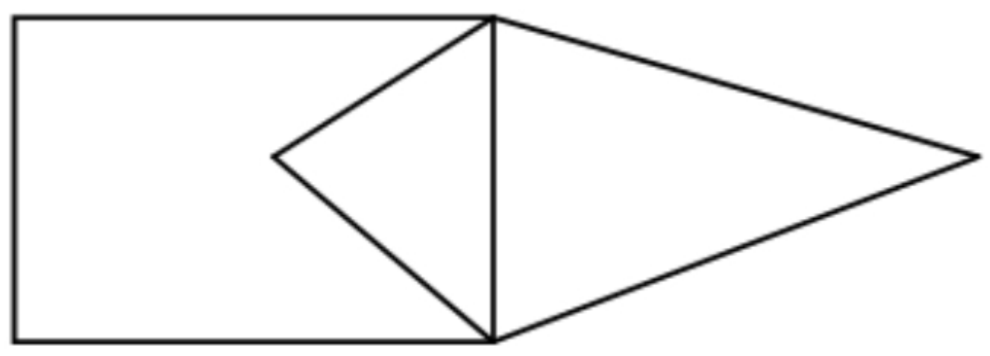
5 a) What are the basic conditions to be satisfied for two graphs to be isomorphic? Are the two graphs below isomorphic? Explain with valid reasons (6)





b) Write any two applications of graphs with sufficient explanation (3)

6 a) Consider the graph G given below: (4)



Define Euler graph. Is G an Euler? If yes, write an Euler line from G.

b) What is the necessary and sufficient condition for a graph to be Euler? And also prove it. (5)

7 a) Define Hamiltonian circuits and paths with examples. Find out the number of edge-disjoint Hamiltonian circuits possible in a complete graph with five vertices (5)

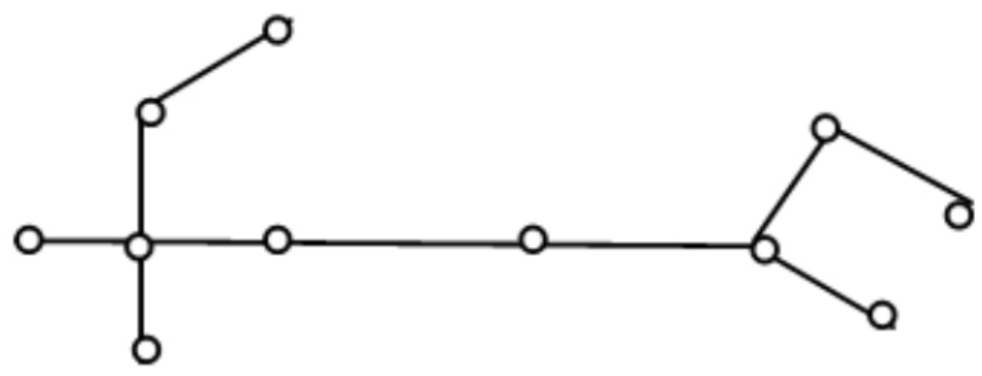
b) State Travelling-Salesman Problem and how *TSP* solution is related with Hamiltonian Circuits? (4)

PART C

Answer all questions, each carries 3 marks.

8 List down any two properties of trees and also prove the theorem: *A graph is a tree if and only if it is a minimally connected.* (3)

9 Consider the tree T, given below (3)



Label the vertices of T appropriately and find the center and diameter ofT

10 Prove the statement: (3)

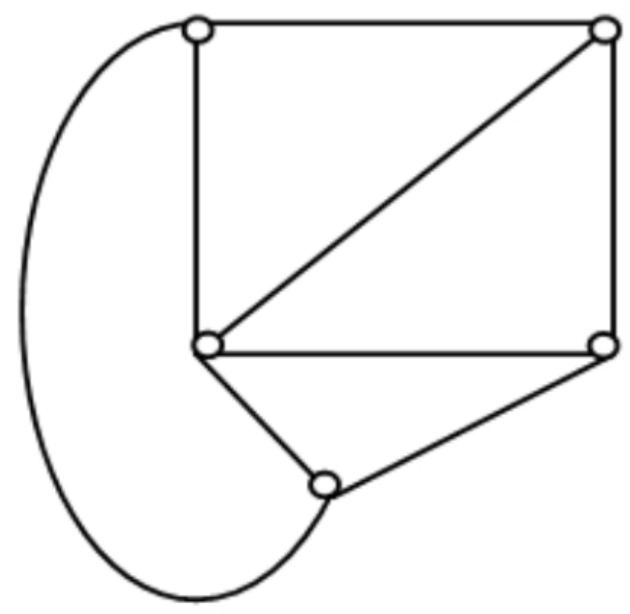
Every cut-set in a connected graph G must also contain at least one branch of every spanning tree of G

11 List down the properties stating the relationship between the edges of graph G and its dual G* (3)

PART D

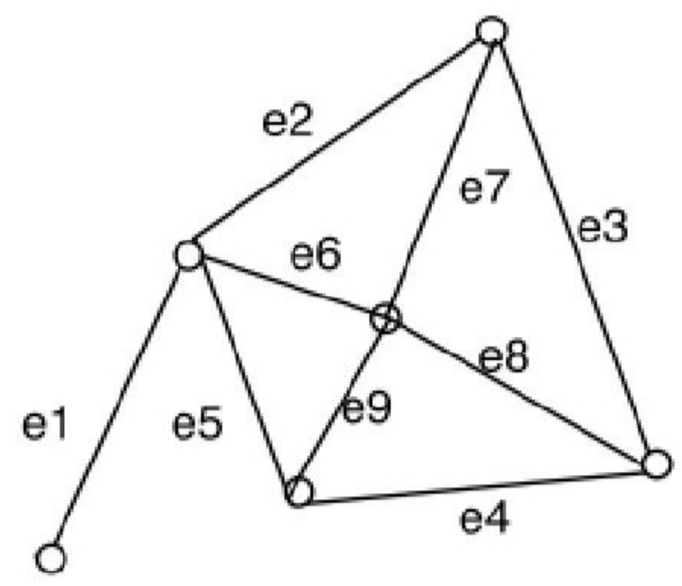
Answer any two full questions, each carries 9 marks.

12 a) Define spanning trees. Consider the graph G given below and obtain any *three* spanning trees from G. Calculate the number of distinct spanning trees possible from a complete graph with *n* vertices. (5)



b) Let G = (V,E) be a connected graph, and let T = (V, S) be a spanning tree of G. Let e = (a, b) be an edge of G *not in* T. Prove that, for any edge f on the path from a to b in T, (V, (S∪{e}) −{f}) is another spanning tree for G (4)

13 a) Define cut set. Find any four cut sets from the graph G given below and also find the edge connectivity of G. (5)



b) Define vertex connectivity and draw a graph with an articulation point. (3)

c) State Euler's Theorem *(formula)*. (1)

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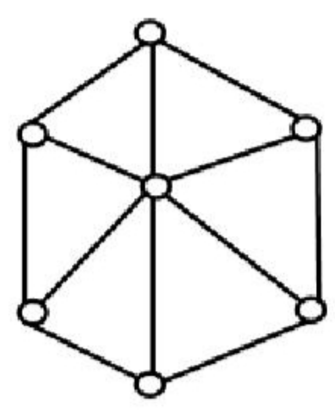
a) Draw two Kuratowski's graphs and also prove that Kuratowsk's first graph is non planar using appropriate inequality. (4)

b) Draw the geometric dual (G*) of the graph G given below and also check whether G and G* are self dual or not, substantiate your answer clearly? (5)

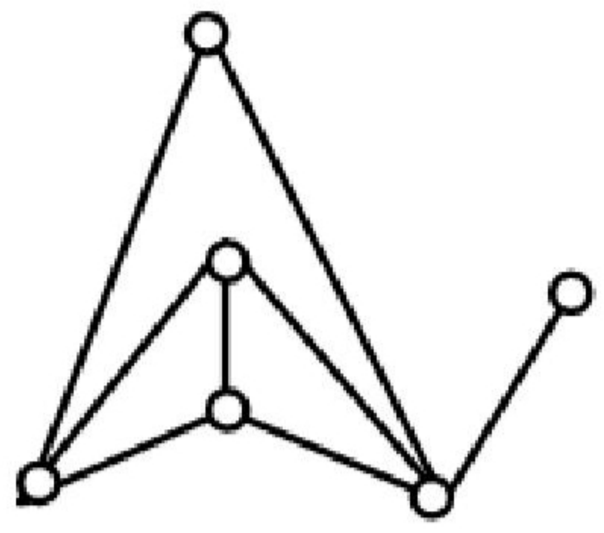


PART E

Answer any four full questions, each carries10 marks.

15 a) List down any four properties of adjacency matrix (4)

b) Construct an adjancy matrix(X) for the following graph and also mention how the concept of edge sequencs is described with X^3 *(no need to find X^3 from X)* (6)



16 a) Prove the theorem: (4)

If A(G) is an incidence matrix of a connected graph G with *n* vertices, the rank of A(G) is *n-1*

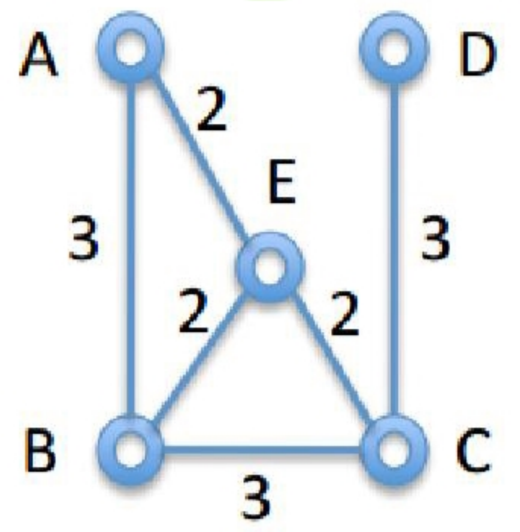
b) Describe with examples the usage of incidence matrix to find two graphs (g1 and g2) are isomorphic. (6)

17 a) Define cut-set matrix with an example and list down any four properties of cut-set matrix (6)

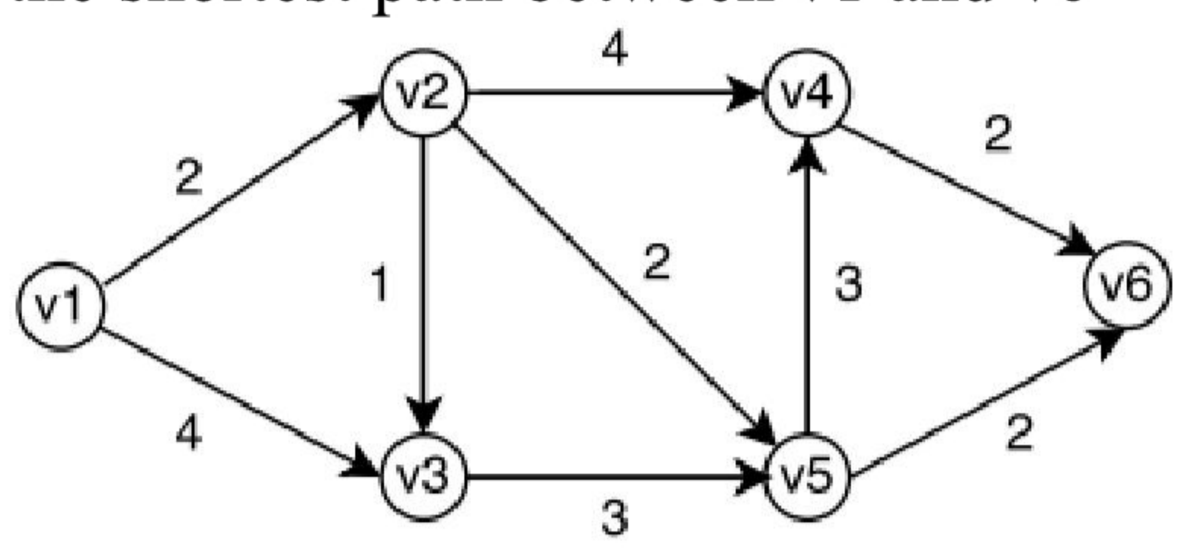
b) If B is a circuit matrix of a connected graph G with *e* edges and *n* vertices, then show that the number of linearly independent rows in B = *e-n+1* (4)

18 a) Draw the flow chart of minimum spanning-tree algorithm. (7)

b) Find MST from the graph given below by simply applying Kruskal's procedure. (3)



19 Write the Dijkstra's shortest path algorithm *(no need to draw flowchart)*. Apply this algorithm to find the shortest path between v1 and v6 (10)



20 Draw the flowchart of *Connectedness and Components* algorithm and also apply this algorithm on any graph (G) with 2 components. (10)

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