Reg. No. $\qquad$ Name: $\qquad$

## APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, DEC 2016

## Course Code: MA201

## Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100
Duration:3. Hours

## PART A

(Answer any two questions)
1.a Show that $u=y^{3}-3 x^{2} y$ is harmonic and hence find its harmonic conjugate.
b Find the image of $\left|\mathrm{z}-\frac{1}{2}\right| \leq \frac{1}{2}$ nder the transformation $=\frac{1}{\mathrm{z}}$. Also find the fixed points of the transformation $\mathrm{w}=\frac{1}{\mathrm{z}}$
2.a Define an analytic function and prove that an analytic function of constant modulus is constant.
b Find the linear fractional transformation that maps $\mathrm{z}_{1}=0, \mathrm{z}_{2}=1, \mathrm{z}_{3}=\infty$ onto $\mathrm{w}_{1}=-1, \mathrm{w}_{2}=-\mathrm{i}, \mathrm{w}_{3}=1$ respectively.
3.a Show that $f(z)=e^{-s} \cos y \quad-\quad i e^{-s} \sin y \quad$ is differentiable everywhere. Find its derivative.
b Find the image of the lines $\mathrm{x}=\mathrm{c}$ and $\mathrm{y}=\mathrm{k}$, where $\mathrm{c} \& \mathrm{kare}$ constants, under the transformation $\mathrm{w}=$ sinz.

## PART B

## (Answer any two questions)

4.a Evaluate $\int_{C} \operatorname{Re}(z) d z$ where $C$ is a straight line from 0 to $1+2 \mathrm{i}$.
b Show that $\int_{0}^{\infty} \frac{d s}{1+s^{4}}=\frac{\mathrm{n}}{2 \sqrt{2}}$
5.a Integrate $\frac{z^{2}-1}{z^{2}}$ counterclockwise around the circle $|z-1-i|=\frac{n}{2}$ by Cauchy's Integral Formula.
b Evaluate $\int_{C} \frac{\mathrm{z}-23}{\mathrm{z}^{2}-4 \mathrm{z}-5} \mathrm{dz}$ where C is $|\mathrm{z}-2-\mathrm{i}|=3.5$ by Cauchy's Residue Theorem
6.a If $f(z)=\frac{1}{z^{2}}$ find the Taylor series that converges in $|z-i|<$ Rand the Laurent's series that converges in $|\mathrm{z}-\mathrm{i}|>\mathrm{R}$.
b Define three types of isolated singularities with an example for each.

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## PART C

## (Answer any two questions)

7.a Solve by Gauss Elimination:

$$
\begin{gather*}
x_{1}-x_{2}+x_{3}=0 \\
-x_{1}+x_{2}-x_{3}=0 \\
10 x_{2}+25 x_{3}=90 \\
20 x_{1}+10 x_{2}=80 \tag{5}
\end{gather*}
$$

b Find the rank. Also find a basis for the row space and column space for

$$
\begin{array}{rrc}
0 & 1 & 0 \\
{[-1} & 0 & -4]  \tag{5}\\
0 & 4 & 0
\end{array}
$$

c Find out what type of conic section the quadratic form $Q=17 x^{2}-30 x y+17 y^{2}=128$ represents and transform it to the principal axes.
8.a Find whether the vectors $\left[\begin{array}{lll}1 & 2-1 & 3\end{array}\right],\left[\begin{array}{ccc}2 & -13 & 2\end{array}\right]$ and $\left[\begin{array}{ccc}-1 & 8-9 & 5\end{array}\right]$ are linearly dependent.
b Show that the matrix $A=\left[\begin{array}{rr}1 & 2 \\ 2 & -2\end{array}\right]$ is symmetric. Find the spectrum.
Diagonalise A $=\left[\begin{array}{rrc}8 & -6 & 2 \\ -6 & 7 & -4\end{array}\right]$
$\begin{array}{lll}2 & -4 & 3\end{array}$
9. a. Determine whether the matrix $\left|\begin{array}{ccc}1 & 0 & -10\end{array}\right|$ ( $/ \sqrt{2} \quad 1 / \sqrt{2} \mid$ is orthogonal?
b. Find the Eigen values and Eigen vectors of $\left.\begin{array}{rll}1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3\end{array}\right]$
c. Define a Vector Space with an example.

