Reg. No.____________ Name:________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, JANUARY 2017

Course Code**: CS 201**

Course Name**: DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)**

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each Question carries 3 marks

1. Show that (A-B) – C=A-(B U C)
2. Show that the set of integers of positive, negative and zero are denumerable.
3. Show that if any five integers from 1 to 8 are chosen, then atleast two of them will have a sum 9.
4. Define:Partition,antisymmetric, Semigroup homomorphism.

PART B

Answer any two questions. Each Question carries 9 marks.

5. a. Prove that every equivalence relation on a set generates a unique partition of the set and the blocks of this partition corresponds to R-equivalence classes. (4.5)

   b.Let X = {1, 2 ……7 }and R = {< X,Y> / X-Y is divisible by 3}. Show that R is an equivalence relation. Draw the graph R. (4.5)

6. a. In how many ways can the letters of the word MONDAY be arranged? How many of them begin with M and end with Y?How many of them do not begin with M but end with Y? (4)

   b. Solve $a_{n+2} – 4\ a_{n+1} + 4a_n = 2^n$, $a_0=1$,$a_1=1$ (5)

7. a. Draw Hasse diagram for D_{100}. Find GLB and LUB for B={10, 20}B={5,10,20,25 } (3)

   b. Let X= {1,2,3} and f,g,h be function from X to X given by f ={(1,2) , (2,3) , (3,1)} g = {(1,2),(2,1),(3,3)} h = { (1,1), (2,2),(3,1) }. Find f og, goh, f o h o g. (3)

   c. In a class of 25 students, 12 have taken Mathematics, 8 have taken Mathematics but not Biology. Find the number of students who have taken Mathematics and Biology and those who have taken Biology but not Mathematics. (3)

PART C

Answer All Questions. Each Question carries 3 marks.

8. Show that inverse of an element a in the group is unique.
9. Show that $(G,+_6)$ is acyclic group where G = {0,1,2,3,4,5}
10. A= {2, 3, 4,6,12,18,24,36} with partial order of divisibility. Determine the POSET is a lattice.
11. Consider the lattice D_{20} and D_{30} of all positive integer divisors of 20 and 30 respectively, under the partial order of divisibility. Show that is a Boolean algebra.

PART D

Answer any two Questions. Each Question carries 9 marks

12. a. Prove that the order of each subgroup of a finite group G is a divisor of the order of the group G. (4.5)

    b. Show that the set{ 0, 1, 2,3,4,5 }is a group under addition and multiplication modulo 6. (4.5)

13. a. Prove that every finite integral domain is a field. (4.5)

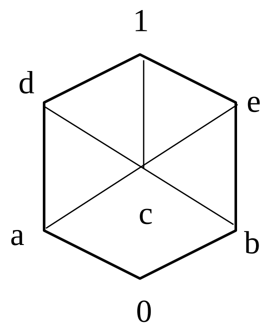
    b. Show that (Z, θ, Θ) is a ring where a θ b = a+b-1 and a Θ b = a+b–ab (4.5)

14. a. Consider the Boolean algebra D_{30}.Determine the following:

    i) All the Boolean sub-algebra of D_{30}.

    ii) All Boolean algebras which are not Boolean sub-algebras of D_{30} having atleast four elements. (4.5)

    b. Consider the Lattice L in the figure. Find the L is distributive and complemented lattice. Also find the complement of a,b,c. (4.5)



PART E

Answer any four Questions. Each Question carries 10 marks

15. a. Without using truth tables, prove the following $(\neg P \vee Q) \wedge (P \wedge (P \wedge Q)) \equiv P \wedge Q$

    b. Show that $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$ is a tautology.

16. a. Convert the given formula to an equivalent form which contains the connectives $\neg$ and $\wedge$ only**:** $\neg (P \leftrightarrow (Q \rightarrow (R \vee P)))$

    b. Show that $\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.

17. a. Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$

    b. Prove the validity of the following argument "If I get the job and work hard, then I will get promoted. If I get promoted, then I will be happy. I will not be happy. Therefore, either I will not get the job or I will not work hard.

18. a. Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)(P(x) \wedge (\exists x)(Q(x)$

    b. Consider the statement "Given any positive integer, there is a greater positive integer". Symbolize this statement with and without using the set of positive integers as the universe of discourse.

19. a. Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.

    b. Prove by mathematical induction that $6^{n+2}+7^{2n+1}$ is divisible by 43 for each positive integer n.

20. Discuss indirect method of Proof. Show that the following premises are inconsistent.

    (i) If Jack misses many classes through illness, then he fails high school.

    (ii) If Jack fails high School, then he is uneducated.

    (iii) If Jack reads a lot of books, then he is not uneducated.

    (iv) Jack misses many classes through illness and reads a lot of books.