**Total Pages: 3**

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**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**
FOURTH SEMESTER B.TECH DEGREE EXAMINATION, JULY 2017

**Course Code: MA204**
**Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)**

Max. Marks: 100 Duration: 3 Hours

***Normal distribution table is allowed in the examination hall.***

**PART A**

***Answer any two questions. Each carries 15 marks***

1 a) A random variable X takes values 0,1, 2 and 3 with probabilities (7)

$$P(X=0)=\frac{8}{15},\ P(X=1)=\frac{1}{3},\qquad P(X=2)=P(X=3)=\frac{1}{15}$$

(i) Find the mean and variance of $X$.

If $Y = 1000 + 300X$ find $P(Y \geq 1500)$ and $E[Y]$

b) In an examination, a candidate has to answer 15 multiple choice questions each of which has 4 choices for the answer. He knows the correct answer to 10 questions and for the remaining 5 questions he chooses the answer randomly. (8)

(i) What is the probability that he answers 13 or more questions correctly?

(ii) What is the mean and variance of the number of correct answers he gives?

2 a) The lifetime of a battery is exponentially distributed. 40% of such batteries do not last longer than 1000 hours. Mr. Kumar purchased such a battery which is already used for 500 hours. What is the probability that it will last another 1000 hours? (5)

b) Find the mean and variance of a random variable $X$ which is uniformly distributed in the interval $[a, b]$ (5)

(c) The monthly salary (in Rs.) of 1000 employees in a factory are normally distributed with mean 20,000 and standard deviation 5000. Estimate the number of employees whose monthly salary will be (i) between 18,000 and 22,000 (ii) less than 18,000? (5)

3 a) Accidents occur at an intersection at a Poisson rate of 2 per day. (7)

(i) What is the probability that there would be no accidents on a given day?

(ii) What is the probability that in January there are at least 3 days (not necessarily consecutive) without any accidents?

b) A printer ink cartridge has a life of X hours under normal usage. The variable X is modelled by the probability density function (8)

$$f(x)=\begin{cases}\frac{k}{x^2}, & x\geq 400,\\ 0, & \text{otherwise.}\end{cases}$$

(i) Find $k$

(ii) Find the probability that such a cartridge has a life of at least 600 hours of normal usage.

(iii) Find the probability that two cartridges will have to be replaced before each has been used for 600 hours.

## PART B

*Answer any two questions. Each carries 15 marks*

4 a) A factory has two outlets to sell its products. The daily sales from the first outlet is uniformly distributed between Rs. 50,000 and 60,000 and from the second outlet is uniformly distributed between 40,000 and 60,000. The sales of the outlets are independent. (7)

(i) What is the probability that the total sales from both the outlets combined is more than Rs.100000.

(ii) If 20% of the amount from the sales is profit, find the expected daily profit from both the outlets combined, and the variance of the profit.

b) A computer generates 100 random numbers uniformly distributed between 0 and 1. Use central limit theorem to find the probability that (8)

i) their sum is 60 or more,

ii) their average is 0.7 or less.

5 a) A random process $X(t)$ is defined by $X(t) = \sin(t + \Theta)$where $\Theta$ is a random variable taking values 0 or $\pi$ with equal probability. Find the mean, autocorrelation and autocovariance of $X(t)$. Is it a wide sense stationary process? (7)

b) Find the power spectral density of a wide sense stationary process $X(t)$ with autocorrelation function $R_X(\tau) = e^{-3|\tau|}$. (8)

6 a) The joint probability distribution of two discrete random variables $X$ and $Y$ is given by (7)

$$p(x,y) = \frac{1}{30}(x+y), x = 0,1,2 \quad y = 0,1,2,3$$

Find the correlation coefficient between $X$and $Y$.

b) Find the autocorrelation function and average power of a wide sense stationary process $X(t)$ with power spectral density given by (8)

$$S_X(\omega) = \begin{cases} 1-\omega, & |\omega| \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

## PART C

*Answer any two questions. Each carries 20 marks*

7 a) The number of enquiries arriving at a call centre is a Poisson process with rate 5 per hour. (10)

i) Find the probability that there would be 3 calls between 10 AM and 11 AM and 4 calls between 2 PM and 4 PM.

ii) A call is categorized as `long' if it lasts more than 10 minutes. The probability that an arriving call is long is 0.2. Find the probability that the time between two consecutive long calls is less than 1 hour.

b) $X_n, n = 0,1,2,\ldots$is a Markov chain on state space $\{1,2,3\}$ with initial probability distribution $P(X_0 = 1) = P(X_0 = 2) = P(X_0 = 3) = 1/3$ and transition probability matrix given by (10)

$$\begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.5 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

Find i) $P(X_2 = 3)$ ii) $P(X_1 = 3, X_2 = 2, X_3 = 2)$ iii) $P(X_2 = 1|X_0 = 2)$

8 a) The table gives the area under the normal probability curve from 0 to certain values $x$. (10)

| $x$ | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
|---|---|---|---|---|---|
| $f(x)$ | 0.1905 | 0.3413 | 0.4332 | 0.4772 | 0.4938 |

Find $f(0.7)$ using suitable form of Newton's interpolation formula.

b) Use Runge-Kutta fourth order method to find $y(0.2)$and $y(0.4)$, given the initial value problem $\frac{dy}{dx} = e^x + y,\ y(0) = 0$ (10)

9 a) Find the probability distribution of the time between two consecutive arrivals in a Poisson process. (5)

b) A machine is in one of two states (i) down (state 0) or (ii) up (state 1). The transition probabilities between the states is given by the following matrix (TPM) (5)

$$\begin{bmatrix} 0.75 & 0.25 \\ 0.40 & 0.60 \end{bmatrix}$$

Find the proportion of time the machine will be up in the long run.

c) Find the real root of $f(x) = e^{2x} - x - 6$ lying between 0 and 1 using Newton-Raphson method. (5)

d) Evaluate $\int_0^1 e^{-x^2/2}\,dx$ numerically using Simpson's rule with a suitable step-size. (5)



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