

10103

Reg. No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, JUNE 2016

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer ALL questions. Each question carries 3 marks

1. Evaluate $\int_0^1 \sinh^2(2x) dx$
2. Check whether the series $\sum_{k=1}^{\infty} \frac{1}{2k-1}$ converges or not.
3. Identify the quadric surface $6x^2 + 3y^2 + 4z^2 = 24$
4. Convert $(2\sqrt{3}, \pi/3, 6)$ from cylindrical to spherical co-ordinates.
5. Find the rate of change of $f(x,y) = xe^y + 5y$ with respect to x at the point $(4,0)$ with y held fixed.
6. If $f(x,y) = x^2y^3 + x^4y$. Find f_{xy}
7. Evaluate $\int_1^9 \left(\left(\frac{t}{2}\right)i + \left(t - \frac{1}{2}\right)j \right) dt$
8. Find $\frac{d\vec{u}}{dt}$ if $\vec{u}(t) = (3t\mathbf{i} + 5t^2\mathbf{j} + 6\mathbf{k}) \cdot (t^2\mathbf{i} + 2t\mathbf{j} + t\mathbf{k})$
9. Sketch the region of integration in $\int_0^1 \int_r^{\sqrt{r}} (x^2 + y^2) dy dx$
10. Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{(x+y+z)} dx dy dz$

PART B

Answer any 2 complete questions each having 7 marks

11. A ball is dropped from a height of h feet and on each bounce rises 75% of the distance it has fallen previously. If it travels a distance of 21 feet what is h ?

12. Use Ratio Test for absolute convergence to find whether the series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2^k}{k!} \text{ converges.}$$

13. Find the Maclaurin's Series for $\frac{1}{1-x}$

Answer any 2 complete questions each having 7 marks

14. For the surface $4x^2 + 9y^2 + 18z^2 = 72$

a. Find the equation of the elliptical trace in the plane

b. $z = \sqrt{2}$

c. Find the length of the major and minor axes of the ellipse.

15. Find $\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln(x^2 + y^2)$

16. Let $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$. Determine the limit of $f(x, y)$ as

(x, y) approaches $(0, 0)$ along the curve C , where C is

(a) $x = 0$ (b) $y = 0$ (c) $y = x$

(d) $y = x^2$ (e) $x = y^2$

Answer any 2 complete questions each having 7 marks

17. Use chain rule to find $\frac{dw}{ds}$ at $s = 1/4$ if

$$w = r^2 - r \tan \theta; \quad r = \sqrt{s}; \quad \theta = \pi s$$

18. Locate all relative extrema and saddle points of $f(x, y) = x^2 + xy - 2y - 3x + 1$

19. The volume V of a right circular cone of radius r and height h is given by $V =$

$$\frac{1}{3} \pi r^2 h. \text{ Suppose that the height decreases from 20 to 19.95 units and the radius}$$

increases from 4 to 4.05 units. Compare the change in volume of the cone with an approximation of this change using a total differential.

Answer any 2 complete questions each having 7 marks

20. The temperature in degree Celsius at a point in the (x, y) plane is

$$T(x, y) = \frac{xy}{1 + x^2 + y^2}$$

Find the rate of change of temperature at $(1, 1)$ in the direction of $(2\hat{i} - \hat{j})$.

21. Find the scalar tangential and normal components of acceleration at time t of a

particle with position vector at time t is $\mathbf{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$

22. Find the equation of the tangent plane and parametric equation for the normal line to the surface $x^2 + y^2 + z^2 = 25$ at $P(3, 0, 4)$

Answer any 2 complete questions each having 7 marks

23. Evaluate $\iint_R \sin \theta \, dA$ where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1 + \cos \theta)$.

24. Find the Jacobian $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ where $x = 4u + v$, $y = u - 2w$,
 $z = v + w$.

25. By changing the order of integration evaluate $\int_0^x \int_x^x \frac{e^{-y}}{y} \, dy \, dx$