A B1A004 (2016)

Reg. No.:.................... Name..............................

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, DEC 2016

(2016 ADMISSION)

Course Code: MA 101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer ALL questions

1 (a) Determine whether the series $\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^{k+2}$ converges and if so, find its sum. (2)

(b) Find the Maclaurin series for the function xe^x (3)

2 (a) If $= x^y$, then find $\frac{\partial^2 z}{\partial x \partial y}$ (2)

(b) Compute the differential dz of the function $z = tan^{-1}(xy)$. (3)

3 (a) Find the domain of $r(t) = \langle\sqrt{5t+1}, t^2\rangle, t_0 = 1$ and $r(t_0)$ (2)

(b) Find the directional derivative of $f(x,y) = e^{2xy}$ at $P\,(5,0\,)$, in the direction of $u = -\frac{3}{5}i + \frac{4}{5}j$ (3)

4 (a) Evaluate $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{1-x^2}\sqrt{1-y^2}}$ (2)

(b) Use double integration to find the area of the plane region enclosed by the given curves $y = \sin x$ and $y = cosx$ for $0 \le x \le \frac{\pi}{4}$ (3)

5 (a) Confirm that $\varphi(x,y,z) = x^2 - 3y^2 + 4z^3$ is a potential function for $F(x,y,z) = 2xi - 6yj + 12\ z^2 k$. (2)

(b) Evaluate $\int F.dr$ where $F(x,y) = sinx\ i + \cos x\ j$ where C is the curve $r(t) = \pi i + tj,\ 0 \le t \le 2$ (3)

6 (a) Using Green's theorem evaluate $\oint ydx + xdy$, where C is the unit circle oriented counter clockwise. (2)

(b) If σ is any closed surface enclosing a volume V and $= 2xi + 2yj + 3zk$, Using Divergence theorem show that $\int_\sigma \int F.n\,dS = 7V$ (3)

PART B

(Each question carries 5 Marks)

Answer any TWO questions

7 Test the nature of the series $\sum_{k=1}^{\infty} \frac{4k^3-6k+5}{8k^7+k-8}$

8 Check whether the series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^k}{k!}$ is absolutely convergent or not.

9 Find the radius of convergence and interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x-5)^k}{5^k}$

Answer any TWO questions

10 If $u = f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

11 A function $f(x,y) = x^2 + y^2$; is given with a local linear approximation $L(x,y) = 2x + 4y - 5$ to $f(x,y)$ at a point P. Determine the point P.

12 Find the absolute extrema of the function $f(x,y) = xy - 4x$ on R where R is the triangular region with vertices (0,0) (0,4) and (4,0).

Answer any TWO questions

13 Evaluate the definite integral $\int_0^1 (e^{2t}i + e^{-t}j + 2\sqrt{t}\,k)dt$.

14 Find the velocity, acceleration, speed, scalar tangential and normal components of acceleration at the given t of $r(t) = 3\,sint\,i + 2cost\,j - sin2t\,k$; $t = \frac{\pi}{2}$

15 Find the equation of the tangent plane and parametric equation for the normal line to the surface $z = 4x^3y^2 + 2y - 2$ at the point (1,-2,10)

Answer any TWO questions

16 Evaluate the integral $\int_0^4 \int_y^4 \frac{x}{x^2+y^2}\,dxdy$ by first reversing the order of integration.

17 Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\,dzdxdy$

18 Find the volume of the solid in the first octant bounded by the co-ordinate

planes and the plane $x + y + z = 1$

PART C

(Each question carries 5 Marks)

Answer any THREE questions

19 Find div F and curl F of $F(x,y,z) = x^2yi + 2y^3zj + 3zk$

20 Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $r = \|xi + yj + zk\|$

21 Find the work done by the force field $F(x,y,z) = (x^2 + xy)\ i + (y - x^2y)j$ on a particle that moves along the curve $C: x = t, y = \frac{1}{t}$,$1 \le t \le 3$

22 Evaluate $\int F.dr$ where $F(x,y) = y\ i - x\ j$ along the triangle joining the vertices (0,0), (1,0), and (0,1).

23 Determine whether $F(x,y) = 4y\ i + 4xj$ is a conservative vector field. If so, find the potential function and the potential energy.

Answer any THREE questions

24 Using Green's theorem evaluate $\oint_C (e^x + y^2)\ dx + (e^y + x^2)\ dy)$ where C is the boundary of the region between $y = x^2$ and $y = 2x$.

25 Evaluate the surface integral $\int\int_\sigma \frac{x^2+y^2}{y}\ dS$ over the surface σrepresented by the vector valued function $r(u,v) = 2cosvi + uj + 2sinvk$, $1 \le u \le 3$, $0 \le v \le \pi$

26 Using Divergence Theorem evaluate $\iint_\sigma F.n\ ds$ where $F(x,y,z) = (x - z)i + (y - x)j + (2z - y)k$. σ is the surface of the cylindrical solid bounded by $x^2 + y^2 = a^2, z = 0$, $z = 1$.

27 Determine whether the vector field $F(x,y,z) = 4(x^3 - x)i + 4(y^3 - y)j + 4(z^3 - z)k$ is free of sources and sinks. If it is not, locate them.

28 Using Stokes theorem evaluate $\int_C F.dr$ where $F(x,y,z) = x^2i + 4xy^3j + y^2xk$.
C is the rectangle: $0 \le x \le 1, 0 \le y \le 3$ in the plane $= y$