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Reg No..... Name:.....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2016

(2015 ADMISSION)

Course Code: MA 101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions. Each question carries 3 marks)

1 Find the derivative of $y = (1 + x \cosh^{-1} x)^2$

2 Test the convergence of $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^2}$

3 Classify the surface $4x^2 + 4y^2 + z^2 + 8y - 4z = 4$

4 Convert the rectangular co-ordinate into spherical co-ordinate of $(2, 2\sqrt{3}, -4)$

5 Prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ where $f = x^2y$.

6 Find the velocity, acceleration and speed of a particle moving along the curve $x = 1+3t,\ y = 3-4t,\ z = 1+3t$ $at = 2$

7 Given $z = e^{xy}$, $x = 2u + v$, $y = \frac{u}{v}$ Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$

8 Find the unit tangent vector and unit normal vector to the curve

$x = e^t \cos t,\ y = e^t \sin t,\ z = e^t$ at $t = 0$.

9 Evaluate $\int_0^3 \int_0^{\sqrt{9-y^2}} 2y\,dx\,dy$

10 Find the area of the region R enclosed between the parabola $y = \frac{x^2}{2}$ and the line $y = 2x$

(10*3=30 Marks)

PART B

(Answer any 2 questions each question carries 7 marks)

11 Find the radius of curvature and interval of curvature of $\sum_{n=1}^{\alpha} \frac{x^n}{2n+3}$

12 Test the convergence of $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + - - - - -$

13 Determine the Taylor's series expansion of f(x) = sinx at x = π/4.

(Answer any 2 questions each question carries 7 marks)

14 Find the nature of domain of the following function

1. $f(x,y) = \sqrt{x^2 - y^2}$
2. $f(x,y) = \ln(x^2 - y)$

15 Show that the function $f(x,y) = \frac{x^3 y}{2x^6 + y^2}$ approaches zero as $(x,y) \to (0,0)$ along the line y = mx.

16 Find the trace of the surface $x^2 + y^2 - z^2 = 0$ in the plane x = 2 and y = 1.

$$x^2 + y^2 - z^2 = 0$$

(Answer any 2 questions each question carries 7 marks)

17 Find the local linear approximation of $f(x,y) = \sqrt{(x^2 + y^2)}$ at (3,4) and compare the error in approximation by L(3.04,3.98) with the distance between the points.

18 Find the relative extrema of $f(x,y) = 3x^2 - 2xy + y^2 - 8y$

19 If $z = e^v, x = 2u + v, \ y = \frac{u}{v}$ Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$

(Answer any 2 questions each question carries 7 marks)

20 If $r(t) = e^t i + e^{-2t} j + tk$

1) Find the scalar tangential and normal component of acceleration at t = 0

2) Find the vector tangential and normal component of acceleration at t = 0.

21 Find the equation of the tangent plane and parametric equations of the normal

line to the surface $z = 4x^3y^2 + 2y - 2$ at the point P (1, -2, 10).

22 Find the directional derivative of $f = x^2y - yz^3 + z$ at (1,-2,0) in the direction of $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$

(Answer any 2 questions each question carries 7 marks)

23 Evaluate $\iint_R y\,dA$ where R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line x+y=5

24 Change the order of integration and evaluate $\int_1^2 \int_y^{y^2} y^2\,dx\,dy$

25 Find the volume bounded by the cylinder $x^2 + y^2 = 4$ the planes $y + z = 3$ and $z = 0$.