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Reg. No. \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**THIRD SEMESTER B.TECH DEGREE EXAMINATION, JULY 2017**

Course Code: **MA 201**Course Name: **LINEAR ALGEBRA AND COMPLEX ANALYSIS.**

Max. Marks :100

Duration: 3 hours

**PART A***Answer any two questions.*

1. (a) Does the limit  $\lim_{z \rightarrow 0} \frac{z}{z}$  exist? If yes find the value. If no, explain why? (8)  
 (b) If  $f(z) = u + iv$  is analytic, prove that  $u = \text{constant}$  and  $v = \text{constant}$  are families of curves cutting orthogonally (7)
2. (a) Find the image of the semi-circle  $y = +\sqrt{4 - x^2}$  under the transformation  $w = z^2$  (7)  
 (b) Find the image of the half-plane  $\text{Re}(z) \geq 2$  under the map  $w = iz$  (8)
3. (a) Find the points, if any, in complex plane where the function  $f(z) = 2x^2 + y + i(y^2 - x)$  is  
 (i) differentiable (ii) analytic. (8)  
 (b) Prove that the function  $u(x, y) = x^3 - 3xy^2 - 5y$  is harmonic everywhere. Also find the harmonic conjugate of  $u$ . (7)

**PART B***Answer any two questions.*

4. (a) Evaluate  $\int_C \bar{z} dz$  where  $C$  is given by  $x = 3t, y = t^2, -1 \leq t \leq 4$ . (8)  
 (b) Show that  $\int_C (2 + z)^2 dz = -\frac{i}{3}$  where  $C$  is any path connecting the points  $-2$  and  $-2 + i$  (7)
5. (a) Evaluate  $\int_C \frac{5z+7}{z^2+2z-3} dz$  where  $C$  is the circle  $|z - 2| = 2$ . (8)  
 (b) Find the Laurent's series expansion of  $\frac{1}{z-z^3}$  in  $1 < |z + 1| < 2$ . (7)
6. (a) Use Cauchy's integral formula to evaluate  $\int_C \frac{z+1}{z^4+2iz^3} dz$  where  $C$  is  $|z| = 1$ . (8)  
 (b) Using Contour integration, evaluate  $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9} dx$  (7)

## PART C

*Answer any two questions.*

7. (a) Using Gauss elimination method, find the solution of the system of equations  
 $x + 2y - z = 3, 3x - y + 2z = 1, 2x - 2y + 3z = 2$  and  $x - y + z = -1$  (7)
- (b) Find the values of  $\mu$  for which the system of equations  $x + y + z = 1, x + 2y + 3z = \mu$  and  $x + 5y + 9z = \mu^2$  will be consistent. For each value of  $\mu$  obtained, find the solution of the system. (7)
- (c) Prove that the vectors (2,3,0), (1,2,0) and (8,13,0) are linearly dependent in  $R^3$ . (6)

8. (a) Find the rank of the matrix  $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$  (7)

- (b) Find the eigen values and eigen vectors of the matrix  $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$  (7)

- (c) Write the canonical form of the quadratic form  $Q(x, y, z) = 3x^2 + 5y^2 + 3z^2 - 2xy + 2xz - 2yz$  and hence show that  $Q(x, y, z) > 0$  for all non-zero values of  $x, y, z$ . (6)

9. (a) Diagonalize the matrix  $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$  and hence find  $A^4$ . (7)

- (b) If 2 is an eigen value of  $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ , without using its characteristic equation,

find the other eigen values. Also find the eigen values of  $A^3, A^T, A^{-1}, 5A, A - 3I$  and  $\text{adj } A$ . (7)

- (c) Show that  $17x^2 - 30xy + 17y^2 = 128$  represents an ellipse. Also find the equations of the major and minor axes of the ellipse in terms of  $x$  and  $y$ . (6)

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