

10251

Reg. No. _____

Name: _____

SECOND SEMESTER B.TECH DEGREE EXAMINATION, JULY 2016

Course Code: MA-102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100

Duration: 3 hrs

PART A*Answer all questions Each carries 3 marks*

- (1) Find the general solution of $y''' - y = 0$
- (2) Find the wronskian of the following $e^{-x} \cos 5x ; e^{-x} \sin 5x$
- (3) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}$
- (4) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x^2$
- (5) Express $f(x) = x$ as a Fourier series in the interval $-\pi < x < \pi$
- (6) Obtain the half range Fourier sine series for the function e^x in $0 < x < 2$
- (7) Form the partial differential equation by eliminating the arbitrary function from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$
- (8) Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$
- (9) Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 3e^{-5x}$
- (10) State the one dimensional wave equation with boundary conditions and initial conditions for solving it
- (11) In the Heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ what does α^2 indicate. State the boundary and initial conditions for solving it
- (12) Find the steady state temperature distribution in a rod of length 25cm, if the ends of the rod are kept at 20°C and 70°C .

PART B*Answer one full question from each module*Module -I

- (13) (a) Solve $y''' - 8y'' + 37y' - 50y = 0$ (6)

A

- (b) Determine all possible solutions to the initial value problem $y' = 1 + y^2, y(0) = 0$ in $|x| < 3, |y| < 2$ (5)

OR

- (14) (a) Find the general solution of $y^{iv} - y''' - 9y'' - 11y' - 4y = 0$ (6)
(b) Determine all possible solutions to the initial value problem $y' = y^{\frac{1}{2}}, y(0) = 0$. (5)

Module - II

- (15) (a) Solve by method of variation of parameters $\frac{d^2y}{dx^2} + y = x \sin x$. (6)
(b) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$. (5)

OR

- (16) (a) Solve $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2 + 2 \log x$. (6)
(b) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = \sin 3x \sin 2x$. (5)

Module - III

- (17) (a) Obtain the Fourier series for the function $f(x)$ given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & 0 \leq x \leq \pi \end{cases} \quad (6)$$

- (b) Obtain the Fourier series to represent the function

$$f(x) = |\sin x|; -\pi < x < \pi \quad (5)$$

OR

- (18) (a) Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \leq x \leq \pi$ (6)

- (b) Find the half range cosine series for the function $f(x) = x^2$ in the range $0 \leq x \leq \pi$ (5)

Module - IV

$$(19) \quad (a) \text{ Solve } \frac{\partial^3 z}{\partial x^3} - 2 \frac{\partial^3 z}{\partial x^2 \partial y} = 5e^{3x} - 7x^2 y. \quad (6)$$

$$(b) \text{ Solve } (x + y)zp + (x - y)zq = x^2 + y^2 \quad (5)$$

OR

$$(20) \quad (a) \text{ Solve } \frac{\partial^3 z}{\partial x^3} - 4 \frac{\partial^3 z}{\partial x^2 \partial y} + 4 \frac{\partial^3 z}{\partial x \partial y^2} = 2 \sin(3x + 2y). \quad (6)$$

$$(b) \text{ Solve } \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \cos 2x \cos 3y. \quad (5)$$

Module - V

(21) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x, t)$. (10)

OR

(22) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $\lambda x(l - x)$, find the displacement of the string at any distance x from one end at any time t . (10)

Module - VI

(23) A bar 10 cm long with insulated sides has its ends A and B maintained at 30°C and 100°C respectively until steady state conditions prevail. The temperature at A is suddenly raised to 20°C and at the same time that of B is lowered to 40°C . Find the temperature distribution in the bar at time t . (10)

OR

(24) A rod of 30cm long has its ends A and B kept at 30°C and 90°C respectively until steady state temperature prevails. The temperature at each end is then suddenly reduced to zero temperature and kept so. Find the resulting temperature function $u(x, y)$ taking $x = 0$ at A. (10)