10250

A

Reg. No. : .... **Name :** ....

SECOND SEMESTER B.TECH. DEGREE EXAMINATION, MAY/JUNE 2016

MA 102 : DIFFERENTIAL EQUATIONS

Max. Marks : 100 Duration : 3 Hours

PART – A

Answer **all** questions and **each** question carries **3** marks.

1. Determine a linearly independent solution of the differential equation $(x^2 + 1) y^{II} - 2xy^{I} + 2y = 0$ if $y_1 = x$ is solution.

2. Solve the differential equation $y^{IV} + 6y^{III} + 9y^{II} = 0$.

3. Find the particular integral of the differential equation $(D^2 - 2D + 1)y = xe^x$.

4. Solve by the method of variation parameters, $(D^2 + 4)y = \tan 2x$.

5. Develop the Fourier series of $f(x) = x^2$ in $-2 \le x \le 2$.

6. Find the Fourier sine series of $f(x) = e^x$ in $0 < x < 1$.

7. Obtain the partial differential equation by eliminating f and g from $z = xf(y) + yg(x)$.

8. Solve the partial differential equation $(y^2 + z^2)p - xyq + xz = 0$.

9. Obtain the solution of the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ using method of separation of variables when the separation constant $k < 0$.

10. Write any two assumptions involved in deriving one dimensional wave equation.

11. Find the steady state temperature distribution in a rod of length 20 cm if the ends of the rod are kept at 10° C and 70° C.

12. Solve $\frac{\partial u}{\partial t} = h\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0, t) = u(1, t) = 0$ for $t > 0$ and $u(x, 0) = 3 \sin n\pi x$, $0 < x < 1$.

(12×3=36 Marks)

P.T.O.

PART – B

Answer **six** questions – **one full** question from **each** Module.

Module – 1

13. a) Reduce to first order and hence solve the ODE

    i) $y^{II} + (y^{I})^3 \cos y = 0$ and

    ii) $2xy^{II} = 3y^{I}$.

    b) Solve the IVP $y^{II} - 2y^{I} + 5y = 0$, $y(0) = -3$, $y^{I}(0) = 1$. **11**

OR

14. a) Show that the functions x and x ln (x) are linearly independent (use Wronskian). Hence form an ODE for the given basis x, x ln (x).

    b) Solve the IV P$y^{II} + 0.2\,y^{I} + 4.01\,y = 0$, $y(0) = 0$, $y^{I}(0) = 2$. **11**

Module – 2

15. a) Solve the differential equation $(D+1)^2 y = x^2 e^x$.

    b) Solve the differential equation $(x^3D^3 + 3x^2D^2 + xD + 1)y = x + \log x$. **11**

OR

16. a) Solve the differential equation $(D^2 + 1)y = x^2e^x + \sin x$.

    b) Solve the differential equation $(x+1)^2 y^{II} + (x+1)y^{I} - y = 2\sin\log(x+1)$. **11**

Module – 3

17. a) Find the Fourier Series of $f(x) = \begin{cases} x & , 0 < x < 1 \\ 1 - x & , 1 < x < 2 \end{cases}$.

    b) Find the Fourier cosine series of $f(x) = x(\pi - x)$ in $0 < x < \pi$. **11**

OR

18. a) Expand $f(x) = e^{-x}$ in $(-l, l)$ as a Fourier Series.

    b) Find the half range sine series of $f(x) = x \sin x$ in $0 < x < \pi$. **11**

Module – 4

19. a) Form the PDE by eliminating a, b, c from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

b) Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-2\frac{\partial^2 z}{\partial y^2}=e^{2x+y}$. **11**

OR

20. a) Solve : $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$.

b) Solve the partial differential equation $\frac{\partial^3 z}{\partial x^3}-4\frac{\partial^3 z}{\partial^2 x\partial y}+4\frac{\partial^3 z}{\partial x\partial y^2}=\cos(2x+y)$. **11**

Module – 5

21. A tightly stretched string of length 'a' with fixed ends is initially in equilibrium position. Find the displacement u(x, t) of the string if it is set vibrating by giving each of its points a velocity $v_0 \sin(\pi x/a)$. **10**

OR

22. A transversely vibrating string of length 'a' is stretched between two points A and B.The initial displacement of each point of the string is zero and the initial velocity at a distance x from A is kx(a – x). Find the form of the string at any subsequent time. **10**

Module – 6

23. Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature zero if the initial temperature is $f(x)=\begin{cases} x & , 0<x<L/2 \\ L-x & , L/2<x<L \end{cases}$ **10**

OR

24. An insulated rod of length L has its ends A and B maintained at 0° C and 100° C respectively until steady state conditions prevails. If B is suddenly reduced to 0° C and maintained at 0° C, then find the temperature in the rod at a distance x from A at time t. **10**