

Reg. No:.....

Name:.....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY



FIRST SEMESTER B.TECH DEGREE EXAMINATION, FEBRUARY 2017

MA101: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

(Answer All Questions and each carries 5 marks)

1. a) Test the convergence of $\sum_{k=1}^{\infty} \frac{99^k}{k!}$ (2)
- b) Test the convergence of $\sum_{k=1}^{\infty} \frac{1}{3^{k+1}}$ (3)
2. a) Find the slope of the sphere $x^2+y^2+z^2=1$ in the y- direction at $(\frac{2}{3}, \frac{1}{3}, \frac{-2}{3})$. (2)
- b) Find the critical points of the function $f(x,y) = 2xy-x^3-y^3$. (3)
3. a) Find the velocity at time $t = \pi$ of a particle moving along the curve
 $\vec{r}(t) = e^t \sin t \, i + e^t \cos t \, j + t \, k$. (2)
- b) Find the directional derivative of $f(x,y) = xe^y - ye^x$ at the point P(0,0) in the direction of $5i - 2j$. (3)
4. a) Change the order of integration in $\int_0^1 \int_{y^2}^{\sqrt{2-y^2}} f(x,y) dx dy$. (3)
- b) Find the area of the region enclosed by $y = x^2$ and $y = x$. (2)
5. a) Find the divergence of the vector field $f(x,y,z) = x^2y \, i + 2y^3z \, j + 3z \, k$. (2)
- b) Find the work done by $\vec{F} = xy \, i + x^3 \, j$ on a particle that moves along the curve $y^2=x$ from (0,0) to (0,1). (3)
6. a) Using Green's theorem to evaluate $\int_C 2xy \, dx + (x^2 + x) \, dy$ where C is the triangle with vertices (0,0), (1,0) and (1,1). (2)
- b) Use Stoke's theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x-2y)i + (y-z)j + (z-x)k$ and C is the circle $x^2+y^2 = a^2$ in the xy plane with counter clockwise orientation looking down the positive z- axis. (3)

PART B

MODULE I (Answer Any Two Questions)

7. a) Test the convergence of the following series (5)

$$\text{i) } \sum_{k=1}^{\infty} \frac{(k+4)!}{4!k!4^k} \quad \text{ii) } \sum_{k=2}^{\infty} \left(\frac{4k-5}{2k+1}\right)^k$$

8. Use the alternating series test to show that the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(k+3)}{k(k+1)}$ converge. (5)

9. Find the Taylor's series of $f(x) = x \sin x$ about the point $x = \frac{\pi}{2}$. (5)

MODULE II (Answer Any Two Questions)

10. Find the local linear approximation L to $f(x,y) = \ln(xy)$ at $P(1,2)$ and compare the error in approximating f by L at $Q(1.01, 2.01)$ with the distance between P and Q . (5)

11. Show that the function $f(x,y) = 2 \tan^{-1}(y/x)$ satisfies the Laplace's equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0. \quad (5)$$

12. Find the relative minima of $f(x,y) = 3x^2 - 2xy + y^2 - 8y$. (5)

MODULE III (Answer Any Two Questions)

13. Find the unit tangent vector and unit normal vector to $\vec{r} = 4 \cos t \mathbf{i} + 4 \sin t \mathbf{j} + t \mathbf{k}$ at $t = \frac{\pi}{2}$. (5)

14. Suppose a particle moves through 3- space so that its position vector at time t is

$$\vec{r} = t \mathbf{i} + t^2 \mathbf{j} + t^3 \mathbf{k}. \text{ Find the scalar tangential component of acceleration at the time } t=1. \quad (5)$$

15. Given that the directional derivative of $f(x,y,z)$ at $(3,-2, 1)$ in the direction of $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ is -5 and that $\|\nabla f(3, -2, 1)\| = 5$. Find $\nabla f(3, -2, 1)$. (5)

MODULE IV (Answer Any Two Questions)

16. Evaluate the integral $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dx dy$ by reversing the order of integration. (5)

17. Evaluate $\int_0^1 \int_0^{y^2} \int_{-1}^z z dx dy$. (5)

18. Find the volume of the solid in the first octant bounded by the co-ordinate planes and the plane $x+2y+z = 6$. (5)

MODULE V (Answer Any Three Questions)

19. Let $\vec{r} = xi + yj + zk$ and let $r = \|\vec{r}\|$ and f be a differentiable function of one variable
show that $\nabla f(r) = \frac{f'(r)}{r} \vec{r}$. (5)
20. Evaluate the line integral $\int_C [-y dx + x dy]$ along $y^2 = 3x$ from (3,3) to (0,0). (5)
21. Show that $\vec{F}(x,y) = (\cos y + y \cos x) i + (\sin x - x \sin y) j$ is a conservative vector field.
Hence find a potential function for it. (5)
22. Show that the integral $\int_C (3x^2 e^y dx + x^3 e^y dy)$ is independent of the path and hence
evaluate the integral from (0,0) to (3,2). (5)
23. Find the work done by the force field $\vec{F} = xy i + yz j + xz k$ on a particle that moves
along the curve $C: \vec{r}(t) = t i + t^2 j + t^3 k$ where $0 \leq t \leq 1$. (5)

MODULE VI (Answer Any Three Questions)

24. Use Green's theorem to evaluate the integral $\int_C (x \cos y dx - y \sin x dy)$ where
 C is the square with vertices (0,0), $(\pi,0)$, (π,π) and $(0,\pi)$. (5)
25. Evaluate the surface integral $\int_\sigma \int z^2 ds$ where σ is the portion of the
cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 3$. (5)
26. Use divergence theorem to find the outward flux of the vector field $\vec{F} = 2x i + 3y j + z^2 k$
across the unit cube $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. (5)
27. Use Stoke's theorem to evaluate the integral $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x-y) i + (y-z) j + (z-x) k$
and C is the boundary of the portion of the plane $x+y+z = 1$ in the first octant. (5)
28. Use Stoke's theorem to evaluate the integral $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = 2z i + 3x j + 5y k$ and
 C is the boundary of the paraboloid $x^2 + y^2 + z = 4$ for which $z \geq 0$ and C is positively
oriented. (5)
